

Data mining

Partitionnement (Clustering)

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```
In [1]: import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
iris = sns.load_dataset('iris')
iris.head()
```

Out[1]:

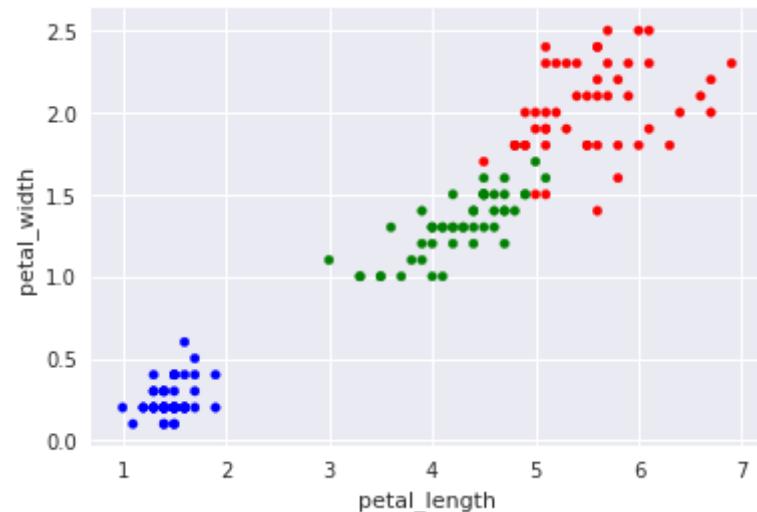
	sepal_length	sepal_width	petal_length	petal_width	species
0	5.1	3.5	1.4	0.2	setosa
1	4.9	3.0	1.4	0.2	setosa
2	4.7	3.2	1.3	0.2	setosa
3	4.6	3.1	1.5	0.2	setosa
4	5.0	3.6	1.4	0.2	setosa

```
In [2]: X_iris = iris.drop(['sepal_length', 'sepal_width', 'species'], axis=1)
y_iris = iris['species']
```

```
In [3]: def get_color(p):
    cols = {'setosa': 'b', 'virginica': 'r', 'versicolor': 'g'}
    return cols[p]

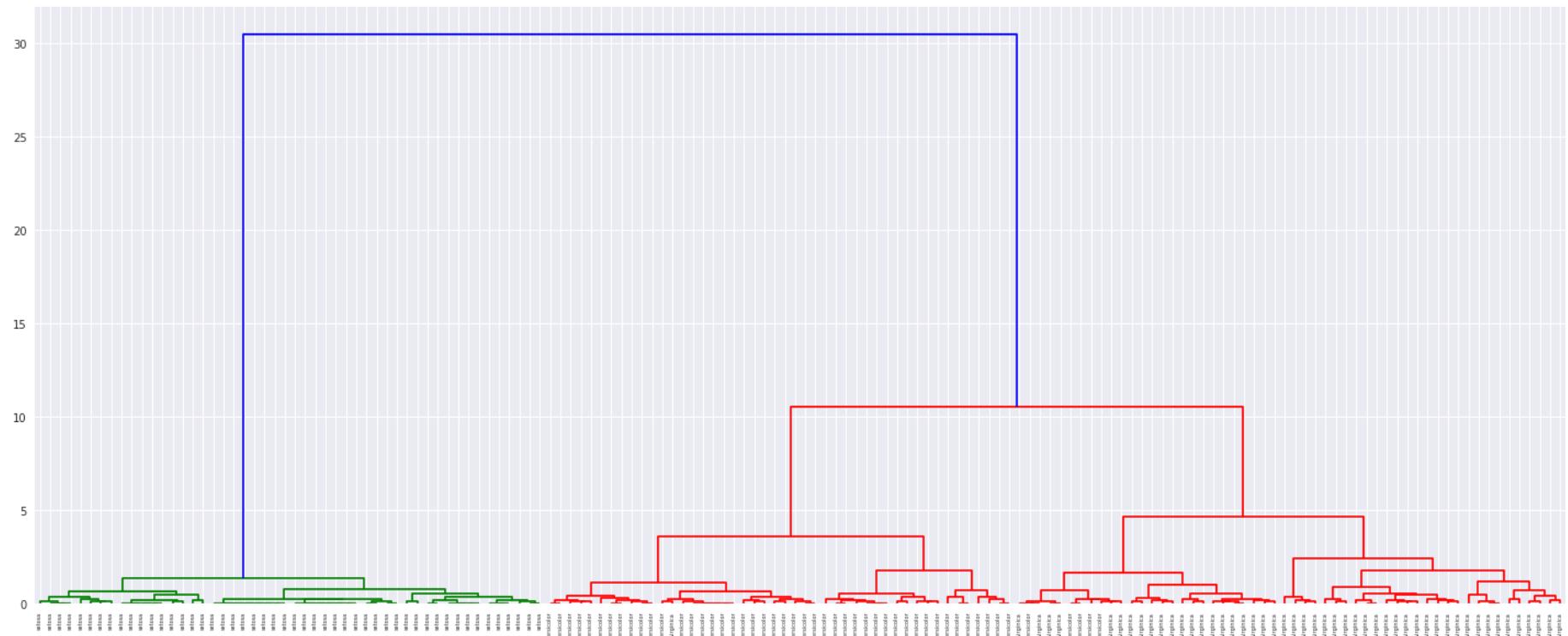
colors = [get_color(p) for p in y_iris]
```

```
In [4]: X_iris.plot.scatter('petal_length', 'petal_width', c=colors)
plt.show()
```



```
In [5]: from scipy.cluster.hierarchy import dendrogram, linkage, fcluster
```

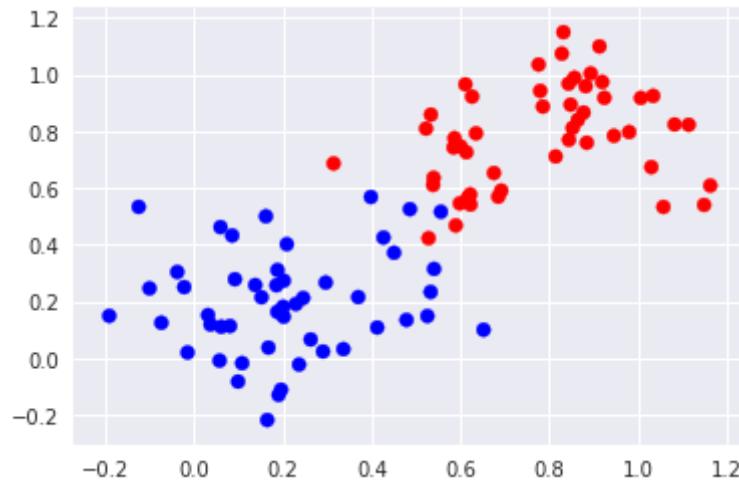
```
In [6]: Z = linkage(X_iris, 'ward')
fig = plt.figure(figsize=(25, 10))
dn = dendrogram(Z, labels=y_iris.values)
```



```
In [ ]:
```

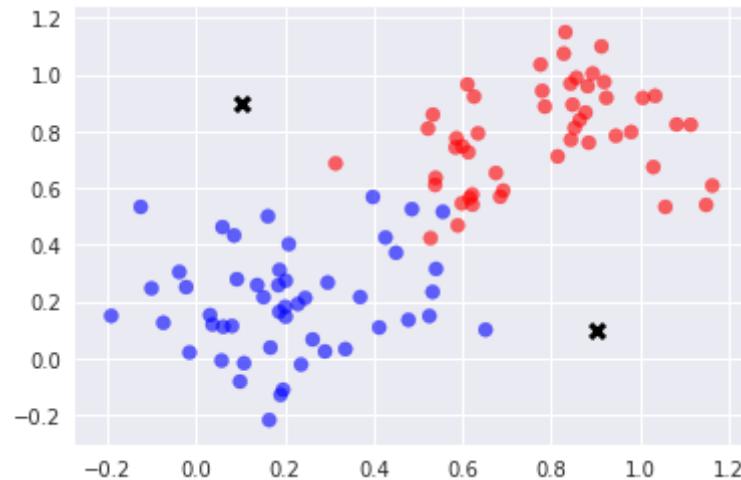
```
In [7]: from sklearn.datasets import make_blobs

X, y = make_blobs(n_samples = 100, n_features=2, centers=[(.2, .2), (.8, .8)], cluster_std=0.2)
colors = ['r' if y_el else 'b' for y_el in y]
plt.scatter(X[:,0], X[:,1], c=colors)
plt.show()
```



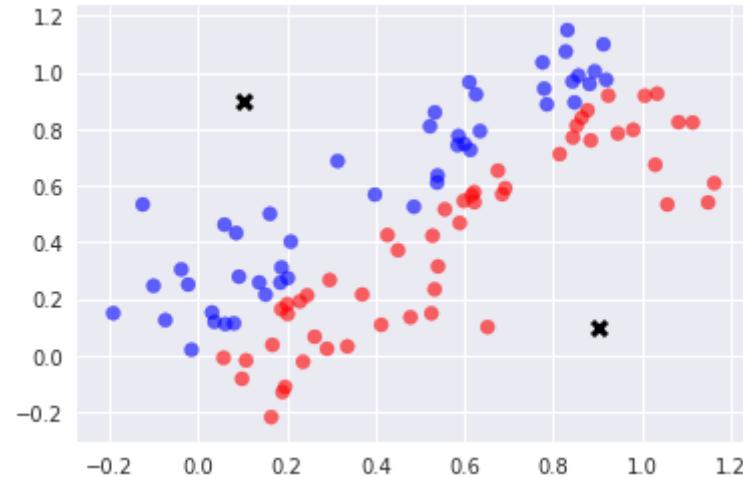
```
In [8]: import numpy as np
```

```
centers = np.array([(0.1, 0.9), (0.9, 0.1)])
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
plt.scatter(centers[:, 0], centers[:, 1], c='k', marker='x', linewidth=3)
plt.show()
```



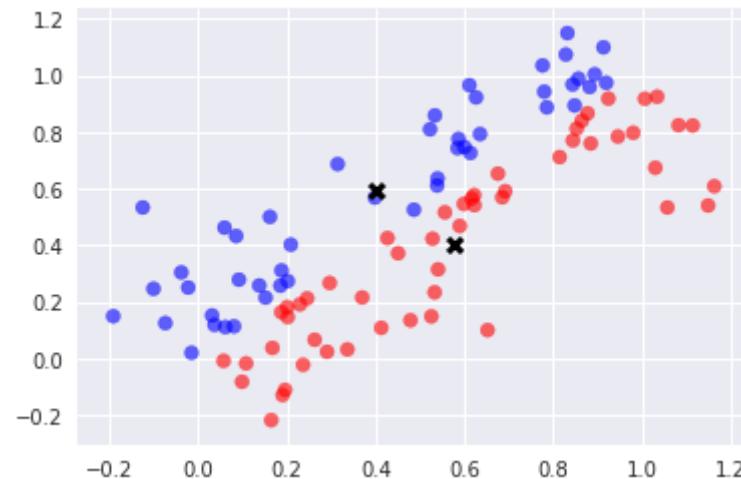
```
In [9]: assigned_clusters = np.array([np.argmin([np.linalg.norm(x - c) for c in centers]) for x in X])
```

```
In [10]: colors = ['r' if y_el else 'b' for y_el in assigned_clusters]
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
plt.scatter(centers[:, 0], centers[:, 1], c='k', marker='x', linewidth=3)
plt.show()
```



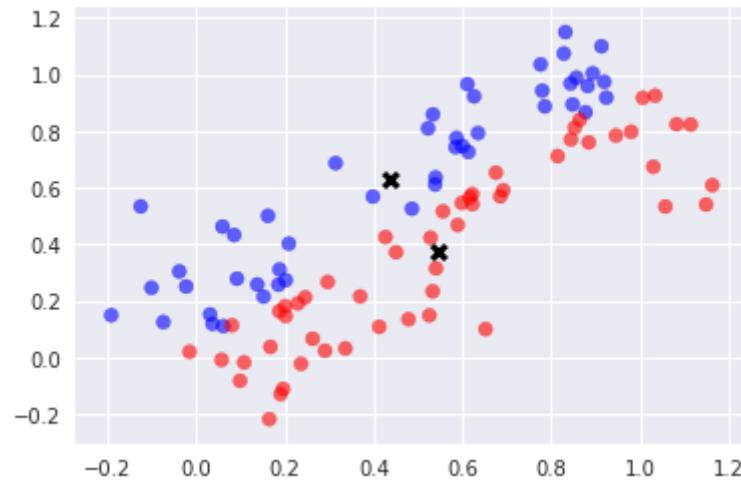
```
In [11]: centers = np.array([np.average(X[assigned_clusters == 0], axis=0),
                           np.average(X[assigned_clusters == 1], axis=0)])
```

```
In [12]: plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
plt.scatter(centers[:, 0], centers[:, 1], c='k', marker='x', linewidth=3)
plt.show()
```

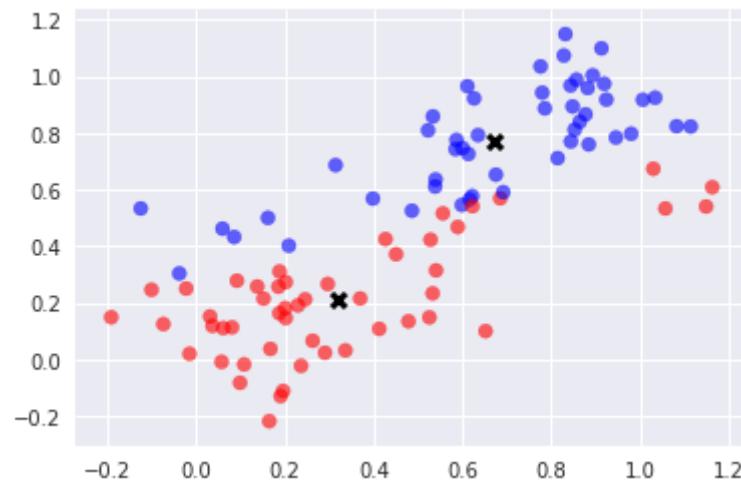


```
In [13]: def kmeans_step(centers, plot=True):
    assigned_clusters = np.array([np.argmin([np.linalg.norm(x - c) for c in centers]) for x in X])
    centers = np.array([np.average(X[assigned_clusters == 0], axis=0),
                      np.average(X[assigned_clusters == 1], axis=0)])
    if plot:
        colors = ['r' if y_el else 'b' for y_el in assigned_clusters]
        plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
        plt.scatter(centers[:, 0], centers[:, 1], c='k', marker='x', linewidth=3)
    return centers
```

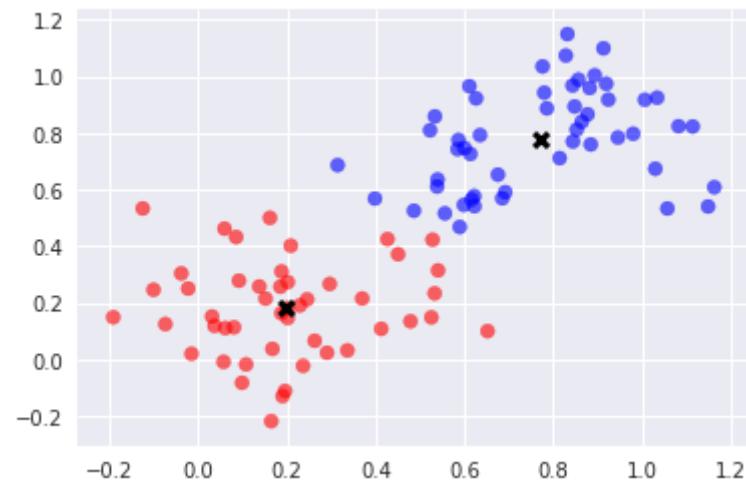
```
In [14]: centers = kmeans_step(centers)
plt.show()
```



```
In [15]: centers = kmeans_step(centers)
plt.show()
```

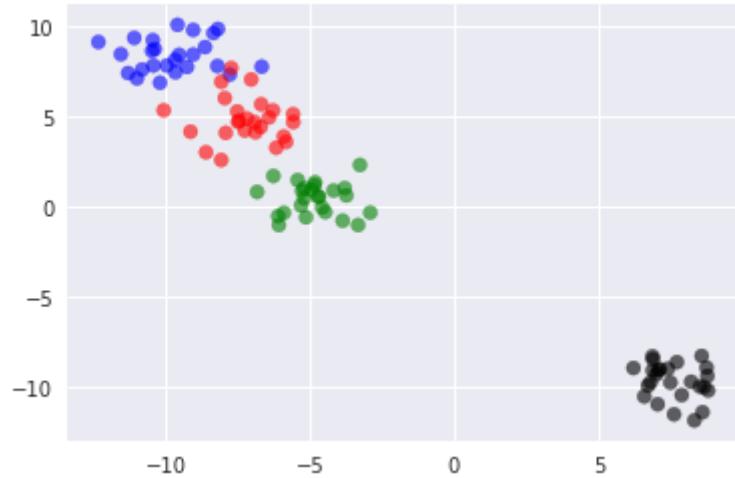


```
In [16]: centers = kmeans_step(centers)  
plt.show()
```

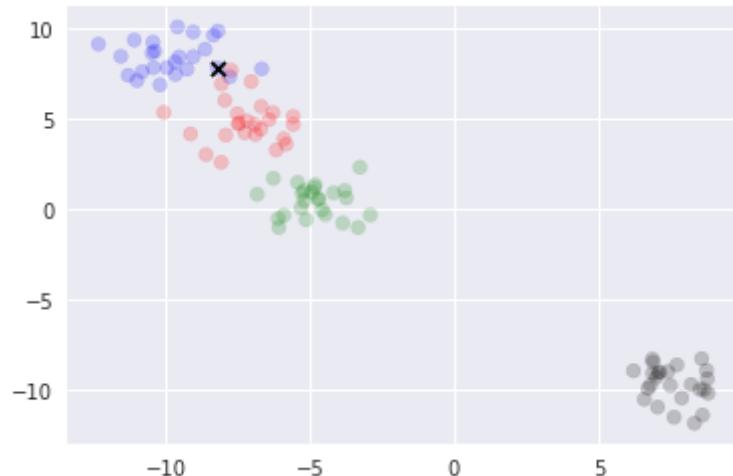


```
In [ ]:
```

```
In [17]: X, y = make_blobs(n_samples = 100, n_features=2, centers=4, cluster_std=1, random_state=12)
colors = [['r', 'g', 'b', 'k'][y_el] for y_el in y]
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
plt.show()
```

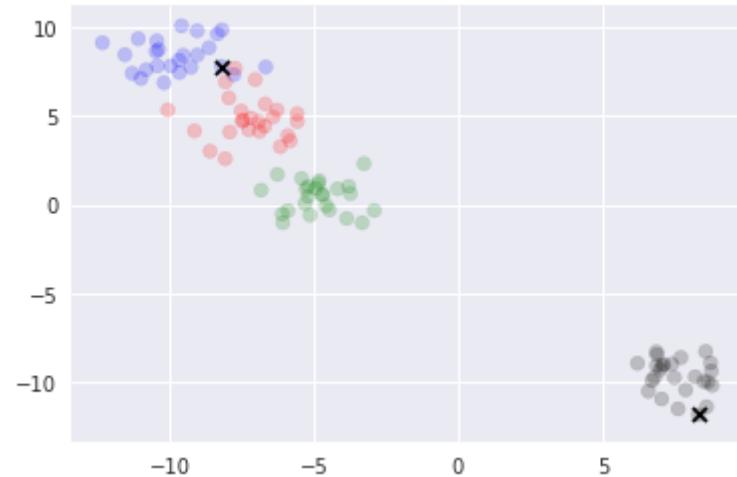


```
In [18]: first_index = np.argmin([np.linalg.norm(x - np.array([-8, 8])) for x in X])
found_centers = np.array([X[first_index]])
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.2)
plt.scatter(found_centers[:, 0], found_centers[:, 1], marker='x', c='k', linewidths=4)
plt.show()
```



```
In [19]: second_index = np.argmax([min([np.linalg.norm(x - c) for c in found_centers]) for x in X])
```

```
In [20]: found_centers = np.vstack([found_centers, X[second_index]])
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.2)
plt.scatter(found_centers[:, 0], found_centers[:, 1], marker='x', c='k', linewidths=4)
plt.show()
```



```
In [21]: third_index = np.argmax([min([np.linalg.norm(x - c) for c in found_centers]) for x in X])

found_centers = np.vstack([found_centers, X[third_index]])
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.2)
plt.scatter(found_centers[:, 0], found_centers[:, 1], marker='x', c='k', linewidths=4)
plt.show()
```



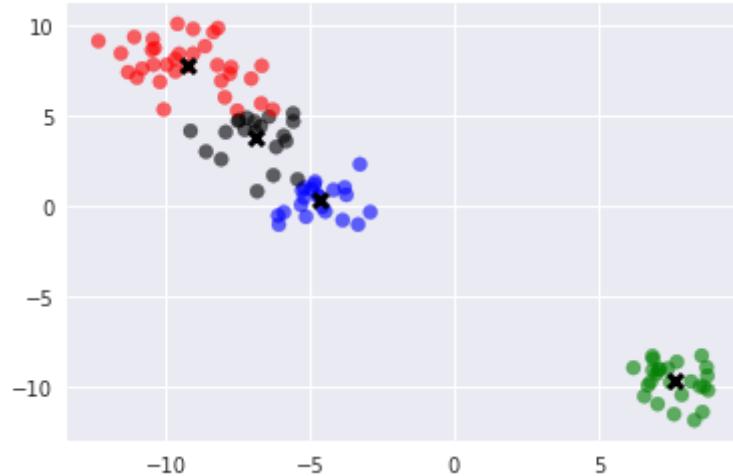
```
In [22]: fourth_index = np.argmax([min([np.linalg.norm(x - c) for c in found_centers]) for x in X])

found_centers = np.vstack([found_centers, X[fourth_index]])
plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.2)
plt.scatter(found_centers[:, 0], found_centers[:, 1], marker='x', c='k', linewidths=4)
plt.show()
```



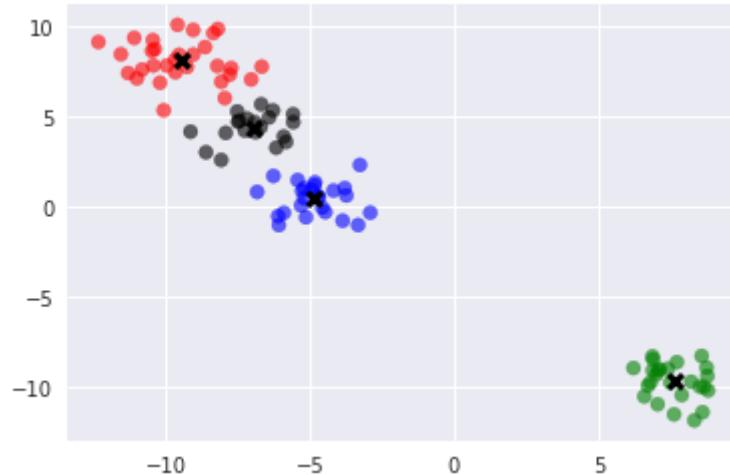
```
In [23]: def kmeans_step(centers, plot=True):
    assigned_clusters = np.array([np.argmin([np.linalg.norm(x - c) for c in centers]) for x in X])
    centers = np.array([np.average(X[assigned_clusters == i], axis=0) for i in range(4)])
    if plot:
        colors =[['r', 'g', 'b', 'k'][y_el] for y_el in assigned_clusters]
        plt.scatter(X[:,0], X[:,1], c=colors, alpha=0.6)
        plt.scatter(centers[:, 0], centers[:, 1], marker='x', c='k', linewidth=3)
    return centers
```

```
In [24]: new_centers = kmeans_step(found_centers)
plt.show()
print(found_centers)
print(new_centers)
print(np.linalg.norm(found_centers-new_centers))
```



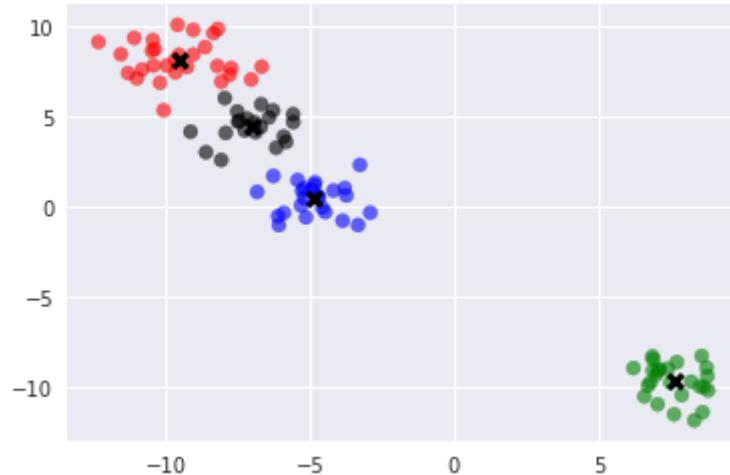
```
[[ -8.20259984  7.78760407]
 [ 8.32880403 -11.81957591]
 [-3.32597193 -1.01290846]
 [-8.07110339  2.58766045]]
 [[-9.23697021  7.82985759]
 [ 7.62646261 -9.66838641]
 [-4.64798564  0.31868486]
 [-6.87708672  3.75444649]]
3.5356093714149694
```

```
In [25]: curr_centers = new_centers  
new_centers = kmeans_step(curr_centers)  
plt.show()  
print(np.linalg.norm(curr_centers-new_centers))
```



0.74174588053625

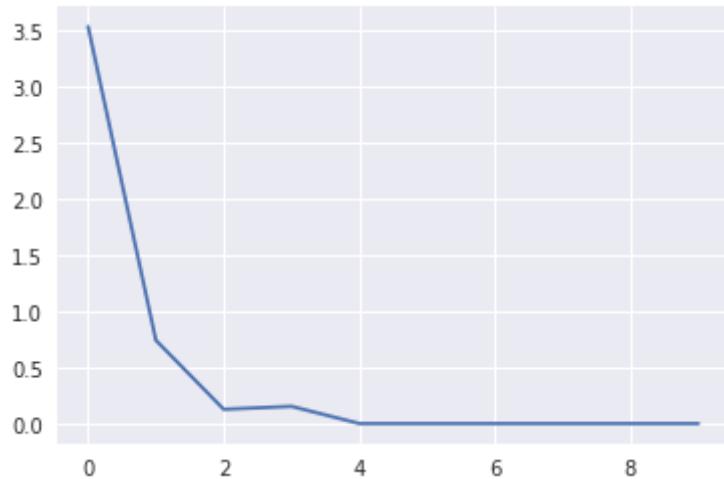
```
In [26]: curr_centers = new_centers
new_centers = kmeans_step(curr_centers)
plt.show()
print(np.linalg.norm(curr_centers-new_centers))
```



0.12686073898808137

```
In [27]: ts = range(10)
errors = []
curr_centers = found_centers
for t in ts:
    new_centers = kmeans_step(curr_centers, plot=False)
    errors.append(np.linalg.norm(curr_centers-new_centers))
    curr_centers = new_centers

plt.plot(ts, errors)
plt.show()
```



```
In [28]: from sklearn.cluster import KMeans

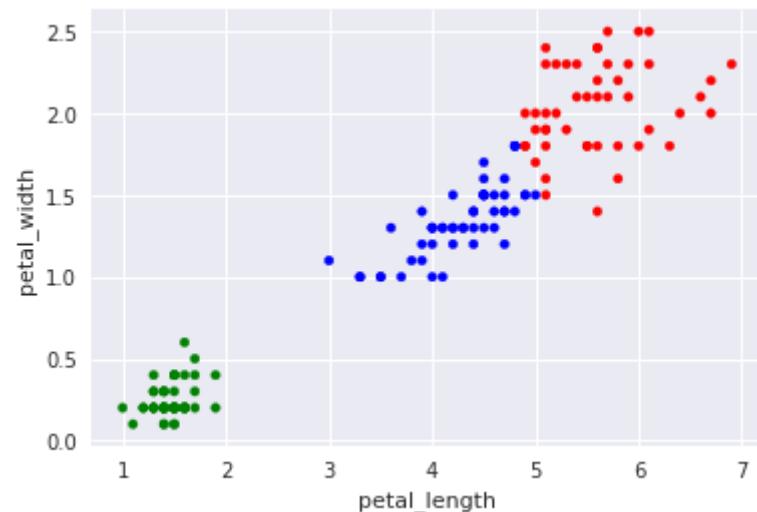
y_pred = KMeans(n_clusters=3).fit_predict(X_iris)
```

In [29]: y_pred

```
In [30]: def get_col(p):
    cols = {0: 'r', 1: 'g', 2: 'b'}
    return cols[p]

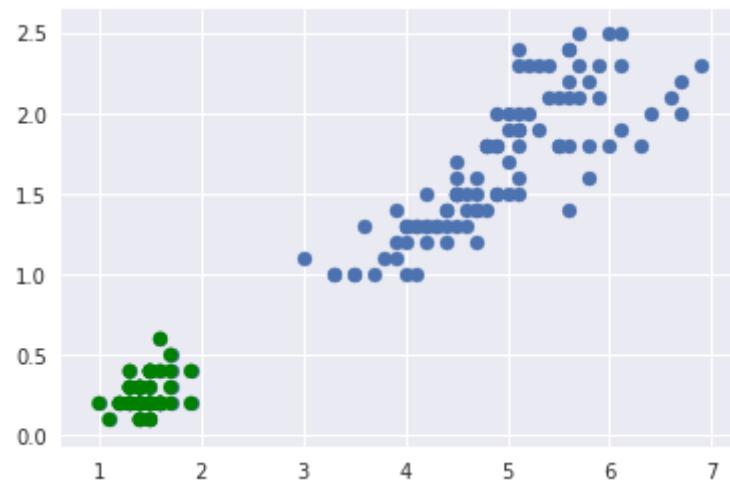
colors = [get_col(y) for y in y_pred]
```

```
In [31]: X_iris.plot.scatter('petal_length', 'petal_width', c=colors)
plt.show()
```

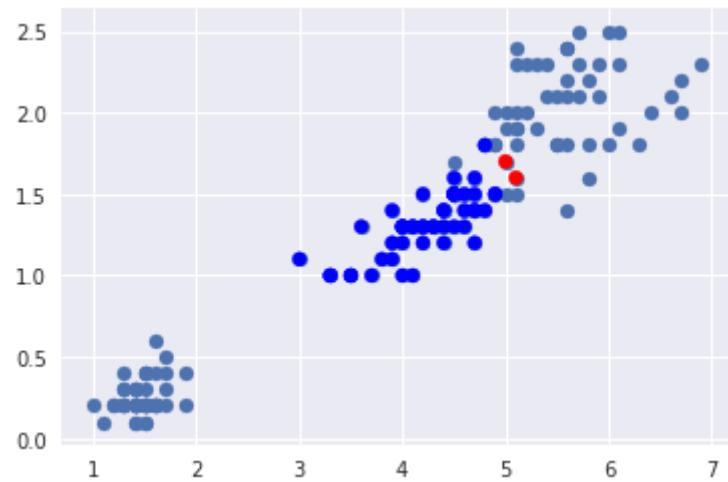


```
In [32]: def show_clustering(left, right):
    plt.scatter(X_iris['petal_length'],
                X_iris['petal_width'])
    plt.scatter(X_iris['petal_length'][left:right],
                X_iris['petal_width'][left:right],
                c=colors[left:right])
    plt.show()
```

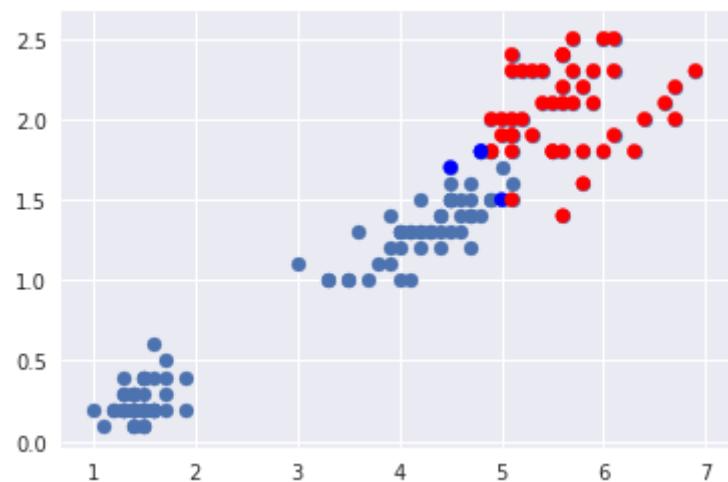
```
In [33]: show_clustering(0, 50)
```



```
In [34]: show_clustering(50, 100)
```

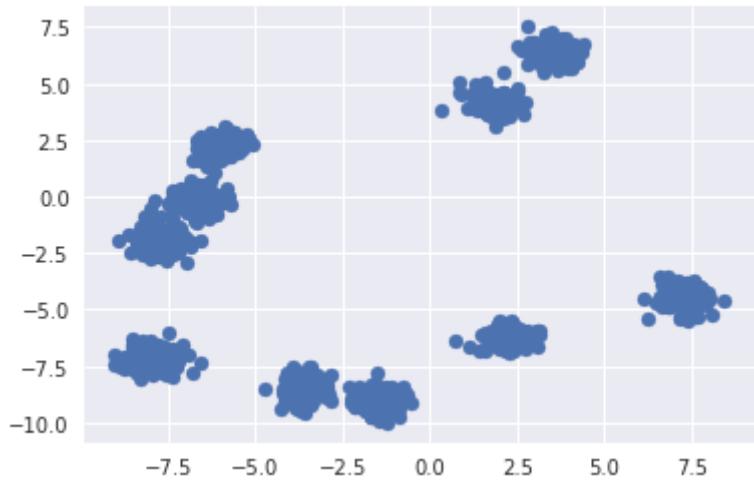


```
In [35]: show_clustering(100, 150)
```



```
In [36]: X, y = make_blobs(n_samples = 1000, n_features=2, centers=10, cluster_std=0.4)
```

```
In [37]: plt.scatter(X[:, 0], X[:, 1])
plt.show()
```



```
In [38]: from sklearn import metrics

kmeans_model = KMeans(n_clusters=3, random_state=1).fit(X)

labels = kmeans_model.labels_
metrics.silhouette_score(X, labels, metric='euclidean')
```

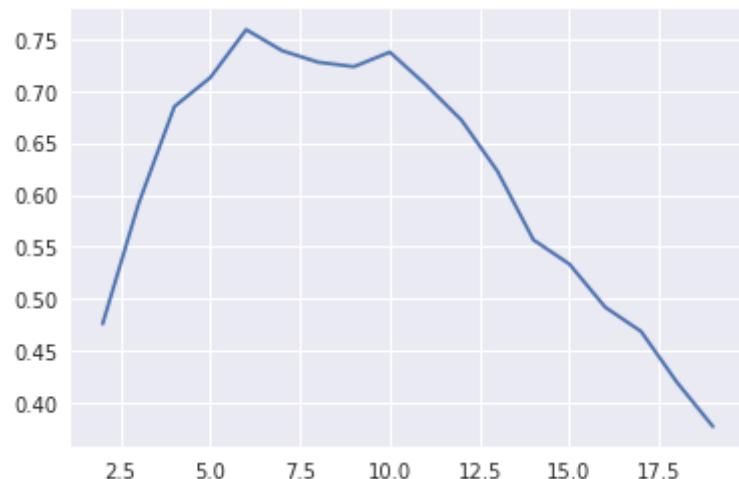
Out[38]: 0.5913044451165838

```
In [39]: def get_silhouette_score(X, d):
    kmeans_model = KMeans(n_clusters=d).fit(X)

    labels = kmeans_model.labels_
    return metrics.silhouette_score(X, labels)
```

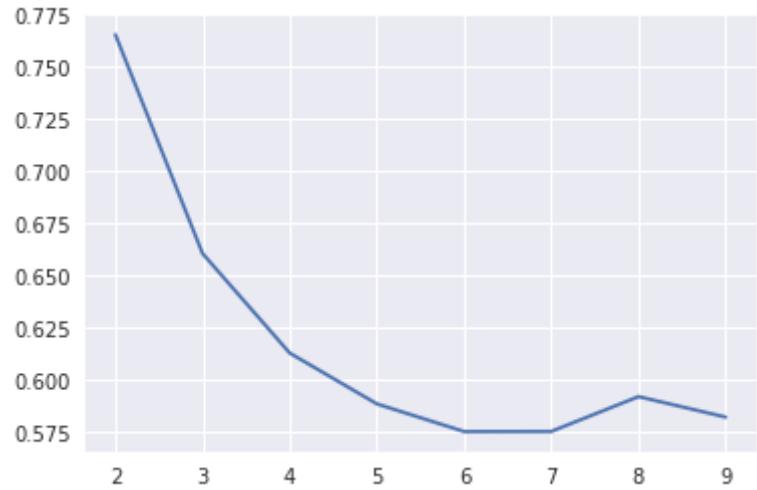
```
In [40]: x = range(2, 20)
s = [get_silhouette_score(X, d) for d in x]

plt.plot(x, s)
plt.show()
```



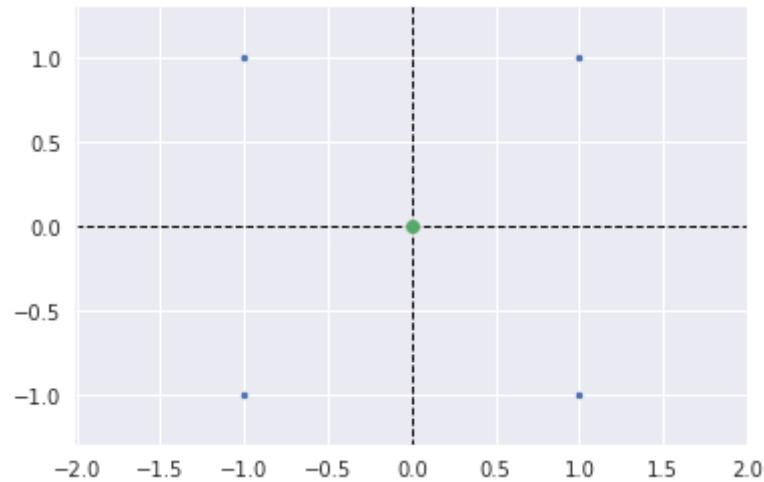
```
In [41]: x = range(2, 10)
s = [get_silhouette_score(X_iris, d) for d in x]

plt.plot(x, s)
plt.show()
```

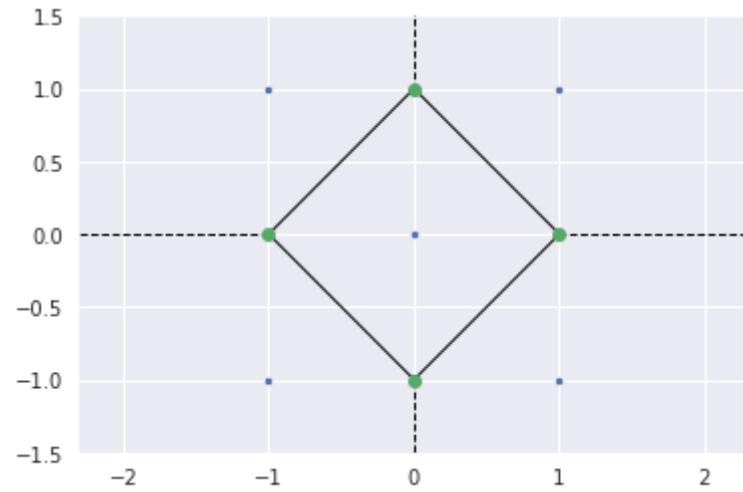


```
In [42]: kmeans_model = KMeans(n_clusters=10).fit(X)
```

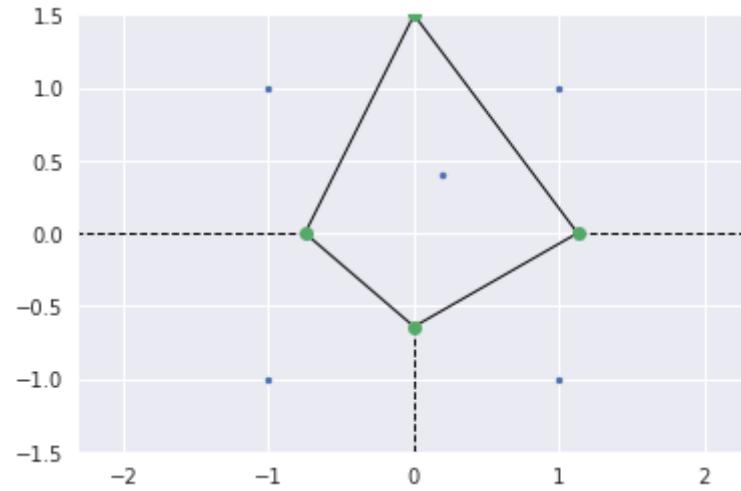
```
In [43]: from scipy.spatial import Voronoi, voronoi_plot_2d
vor = Voronoi([(−1, −1), (−1, 1), (1, −1), (1, 1)])
fig = voronoi_plot_2d(vor)
plt.axis('equal')
plt.xlim((−1.3, 1.3))
plt.ylim((−1.3, 1.3))
plt.show()
```



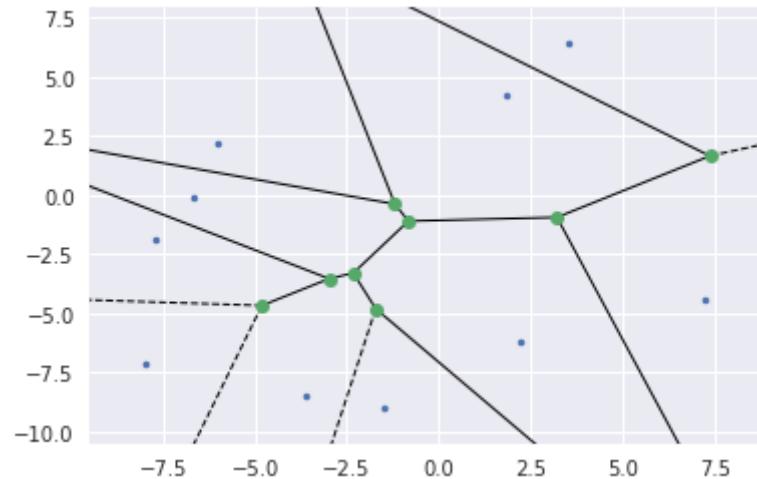
```
In [44]: from scipy.spatial import Voronoi, voronoi_plot_2d
vor = Voronoi([(−1, −1), (−1, 1), (1, −1), (1, 1), (0, 0)])
fig = voronoi_plot_2d(vor)
plt.axis('equal')
plt.xlim((-1.5, 1.5))
plt.ylim((-1.5, 1.5))
plt.show()
```



```
In [45]: from scipy.spatial import Voronoi, voronoi_plot_2d
vor = Voronoi([(−1, −1), (−1, 1), (1, −1), (1, 1), (0.2, 0.4)])
fig = voronoi_plot_2d(vor)
plt.axis('equal')
plt.xlim((−1.5, 1.5))
plt.ylim((−1.5, 1.5))
plt.show()
```



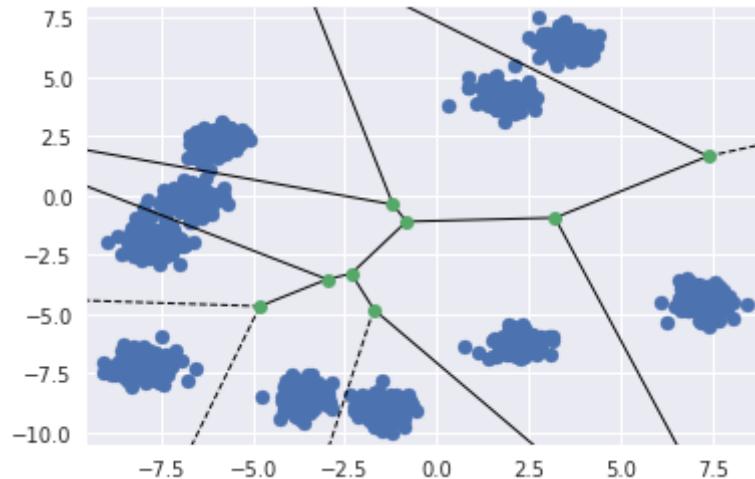
```
In [46]: from scipy.spatial import Voronoi, voronoi_plot_2d  
vor = Voronoi(kmeans_model.cluster_centers_)  
fig = voronoi_plot_2d(vor)  
plt.show()
```



```
In [47]: fig, ax = plt.subplots()
ax.scatter(X[:, 0], X[:, 1])

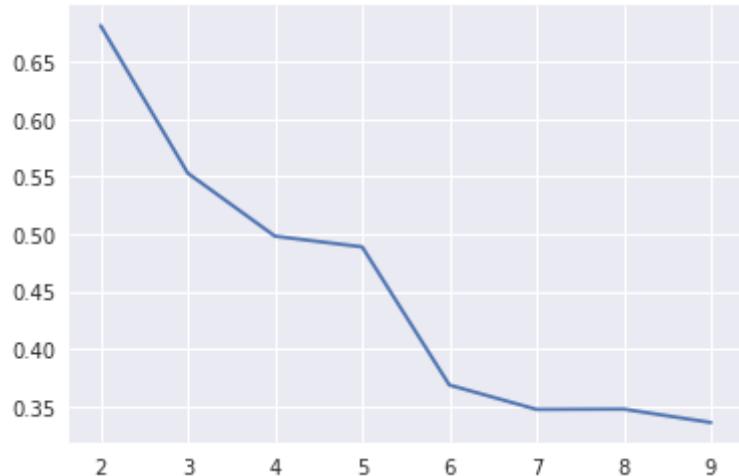
voronoi_plot_2d(vor, ax=ax)
plt.show()
```

/home/malchiodi/anaconda3/lib/python3.6/site-packages/scipy/spatial/_plotutils.py:20: MatplotlibDeprecationWarning: The ishold function was deprecated in version 2.0.
was_held = ax.ishold()



```
In [48]: X_iris_4d = iris.drop(['species'], axis=1)
x = range(2, 10)
s = [get_silhouette_score(X_iris_4d, d) for d in x]

plt.plot(x, s)
plt.show()
```



```
In [49]: from sklearn.datasets import fetch_olivetti_faces
faces_dataset = fetch_olivetti_faces(shuffle=True, random_state=0)
faces = faces_dataset.images.reshape(len(faces_dataset.images), -1)
```

```
In [50]: def plot_faces(faces):
    fig = plt.figure(figsize=(10, 8))
    for i in range(20):
        sub = plt.subplot(4, 5, i + 1)
        sub.imshow(faces[i].reshape(64, 64), cmap='Greys_r')
        plt.xticks(())
        plt.yticks(())

    plt.tight_layout()
    plt.show()

plot_faces(faces)
```



In [51]: `faces[0]`

Out[51]: `array([0.6694215 , 0.6363636 , 0.6487603 , ..., 0.08677686, 0.08264463, 0.07438017], dtype=float32)`

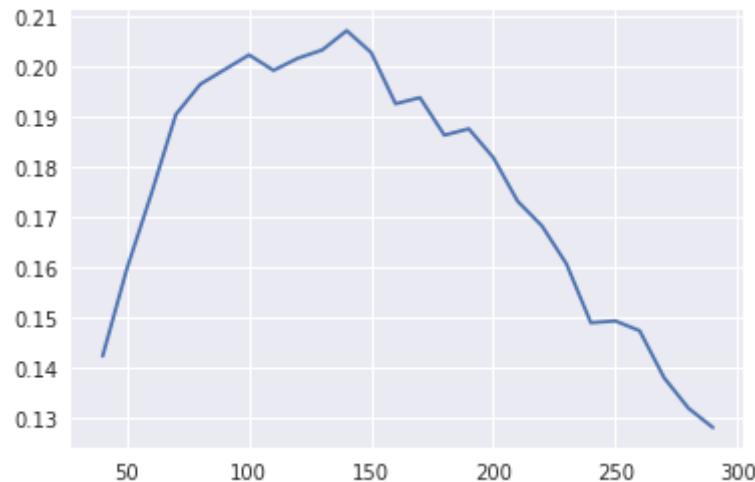
```
In [52]: len(faces)
```

```
Out[52]: 400
```

```
In [53]: x = np.arange(40, 300, 10)
s = [get_silhouette_score(faces, d) for d in x]

plt.plot(x, s)
plt.show()
```

```
/home/malchiodi/anaconda3/lib/python3.6/site-packages/sklearn/metrics/pairwise.py:257: RuntimeWarning: invalid value encountered in sqrt
    return distances if squared else np.sqrt(distances, out=distances)
```



```
In [54]: kmeans_model = KMeans(n_clusters=150).fit(faces)
```

```
/home/malchiodi/anaconda3/lib/python3.6/site-packages/sklearn/metrics/pairwise.py:257: RuntimeWarning: invalid value encountered in sqrt
    return distances if squared else np.sqrt(distances, out=distances)
```

```
In [55]: for f in faces[kmeans_model.labels_ == 1]:
    fig = plt.figure(figsize=(2, 2))
    plt.imshow(f.reshape(64, 64), cmap='Greys_r')
    plt.xticks(())
    plt.yticks()
```



```
In [56]: def show_cluster(c):
    for f in faces[kmeans_model.labels_ == c]:
        fig = plt.figure(figsize=(2, 2))
        plt.imshow(f.reshape(64, 64), cmap='Greys_r')
        plt.xticks(())
        plt.yticks()
```

In [57]: `show_cluster(0)`



```
In [58]: show_cluster(9)
```



```
In [59]: show_cluster(93)
```



```
In [60]: kmeans_model = KMeans(n_clusters=50).fit(faces)
```

```
In [61]: show_cluster(2)
```



```
In [62]: len(faces)
```

Out[62]: 400

```
In [64]: set(faces_dataset.target)
```

```
Out[64]: {0,  
          1,  
          2,  
          3,  
          4,  
          5,  
          6,  
          7,  
          8,  
          9,  
          10,  
          11,  
          12,  
          13,  
          14,  
          15,  
          16,  
          17,  
          18,  
          19,  
          20,  
          21,  
          22,  
          23,  
          24,  
          25,  
          26,  
          27,  
          28,  
          29,  
          30,  
          31,  
          32,  
          33,  
          34,  
          35,  
          36,  
          37,
```

```
38,  
39}
```

```
In [65]: train, test = faces[:300], faces[300:]  
kmeans_model = KMeans(n_clusters=150).fit(train)
```

```
/home/malchiodi/anaconda3/lib/python3.6/site-packages/sklearn/metrics/pairwise.py:257: RuntimeWarning: invalid value encountered in sqrt  
    return distances if squared else np.sqrt(distances, out=distances)
```

```
In [66]: def show_image(i):  
    plt.figure(figsize=(2, 2))  
    plt.imshow(test[i].reshape(64, 64), cmap='Greys_r')  
    plt.xticks(())  
    plt.yticks(())  
  
def show_cluster(c):  
    for f in train[kmeans_model.labels_ == c]:  
        fig = plt.figure(figsize=(2, 2))  
        plt.imshow(f.reshape(64, 64), cmap='Greys_r')  
        plt.xticks(())  
        plt.yticks()
```

```
In [67]: cluster = kmeans_model.predict([test[3]])[0]  
show_cluster(cluster)
```



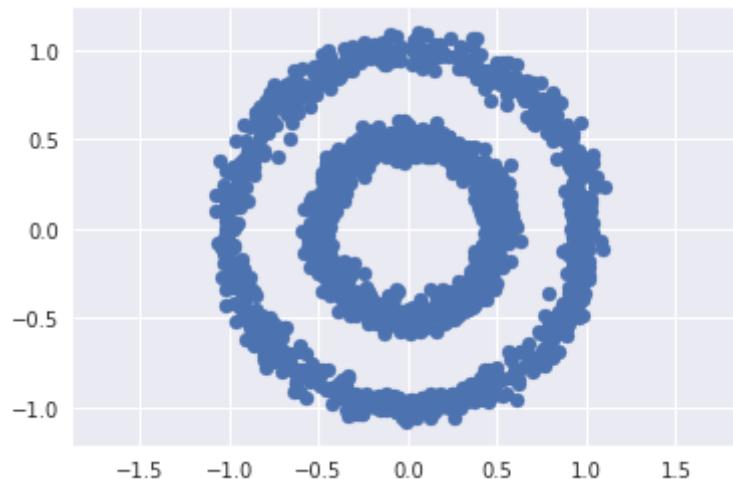
```
In [68]: show_image(3)
```



```
In [69]: from sklearn.datasets import make_circles  
  
n_samples = 1500  
noisy_circles = make_circles(n_samples=n_samples, factor=.5,  
                             noise=.05)
```

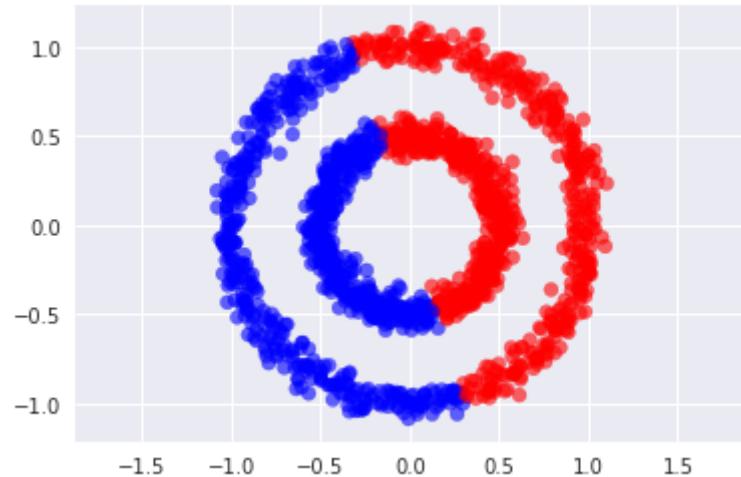
```
In [70]: X, y = noisy_circles
```

```
In [71]: plt.scatter(X[:, 0], X[:, 1])  
plt.axis('equal')  
plt.show()
```

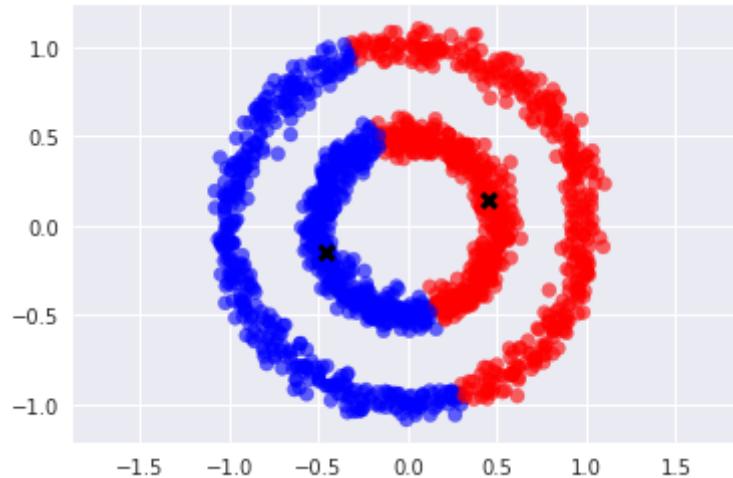


```
In [72]: kmeans_model = KMeans(n_clusters=2, random_state=1).fit(X)  
  
labels = kmeans_model.labels_
```

```
In [73]: colors = ['r' if y else 'b' for y in labels]
plt.scatter(X[:, 0], X[:, 1], c=colors, alpha=0.6)
plt.axis('equal')
plt.show()
```



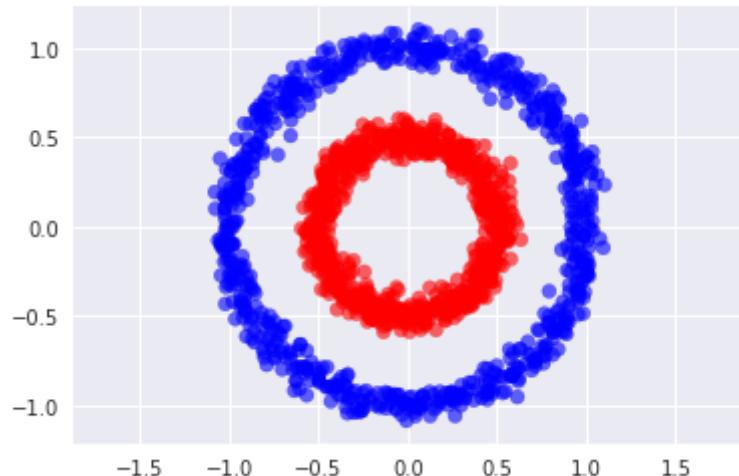
```
In [74]: centers = kmeans_model.cluster_centers_
plt.scatter(X[:, 0], X[:, 1], c=colors, alpha=0.6)
plt.scatter(centers[:, 0], centers[:, 1], c='k', marker='x', linewidth=3)
plt.axis('equal')
plt.show()
```



```
In [75]: from sklearn.cluster import DBSCAN
sc_model = DBSCAN(eps=.1).fit(X)
labels = sc_model.labels_
```

```
In [76]: colors = ['r' if y else 'b' for y in labels]

plt.scatter(X[:, 0], X[:, 1], c=colors, alpha=0.6)
plt.axis('equal')
plt.show()
```



```
In [77]: sc_model = DBSCAN(eps=6, min_samples=2).fit(faces)
```

```
In [78]: sc_model.labels_
```

```
Out[78]: array([ 0,  1, -1,  2,  3, -1, -1,  4,  5,  6,  7,  8, -1,  9, -1, 10, 11,
   12, 13, 14, 15, -1, 16,  9, 15, -1, -1, 17, 18, 19, 15,  1, 20, 21,
   22, -1, -1, 15, 15, 23, 19, 24, -1, -1, 11, 10, 25, -1, -1, 26, -1,
   5, 11, 15, 27, -1, 28, 29, -1, 30, 31, -1, -1, -1, 14, 25, 12, -1,
  32, 33, -1,  4, 22, 34, 19, -1, 35,  1, 35, 24, 20, -1, 10, 19, 36,
 -1, 37, 25, 38, -1, 39, 40, 10, -1, 14,  5,  0, 38, -1, 35, 41, -1,
 -1, 42, 18, -1, 21, -1, 32, 11, -1, -1, 11, 34, 35,  6, -1, 43, 26,
 -1, 44, 21, -1, 26, 25, 12, 34, -1, -1, 45, -1, 23, -1,  3,  6, 19,
 -1,  2, 24, 29, 46, 47,  7, 48,  4, 49, -1,  3, 50, 48, 51, 52, 27,
 -1, -1, -1, 53, -1,  2, 11, 27,  8, 54, 55, 56, 54, 50, -1, 21, -1,
 -1, 57, 58, 43, 50, -1, 49, 59, -1, 60, 21, -1, -1, 39, 54,  6, 61,
 62, 63, -1, 28, 24, 64, -1, -1,  6, -1,  6, 25, 65,  0, 50,  6, 66,
 -1, 36, 26, -1,  5, 43, -1, 67, 51, 30,  3, 16, 44, 48, 45, 39, 39,
 -1, 59, 17, -1, -1,  3, 43, -1, -1, -1, -1, 39, 39, -1, 46, -1, -1,
 21, 62, 38, -1, 40, 47,  5, 55, -1,  5, -1, 42, 29,  4, -1, 19, 24,
  5, 33,  0, 50, -1,  3, 36, 29, 21, 36, -1, 66, -1, 31, 47,  1, 25,
 68, 62, 69, 21, 41, -1, 11, 21,  4, 54, 39, 10, 59, 53, 28, 13, -1,
 -1, 51, 37, 21, 22,  1,  8, 47, 67, 59, -1, 58, 51, -1, 39, 15, 54,
 -1, -1, -1, 69, -1, -1, 36, 30, 13, -1, 65,  5, 12, 62, -1, -1, 60,
  4, 24,  5, 21, 52, 68, -1, -1, 21,  4, -1, 11, 10, 24, 40, 69, 59,
 19, 63, -1, 64, -1, 10, 68, -1, 11, 61, 47, 56, 42,  3, 58, -1, -1,
 -1, -1, 25, 57, 63, -1, 18, -1, 15, -1, 51, 51, -1,  3, -1, -1, 31,
 -1, -1, 45, 51, 39, 15, 39, -1, 70, -1, 21, -1, 21, 11,  3, -1,  6,
 -1, -1, 70, -1, 24, 14, -1, 30, 15])
```

```
In [79]: def show_cluster(c):
    for f in faces[sc_model.labels_ == c]:
        fig = plt.figure(figsize=(2, 2))
        plt.imshow(f.reshape(64, 64), cmap='Greys_r')
        plt.xticks(())
        plt.yticks()
```

In [80]: `show_cluster(30)`



```
In [81]: show_cluster(-1)
```

```
/home/malchiodi/anaconda3/lib/python3.6/site-packages/matplotlib/pyplot.py:537: RuntimeWarning: More than  
20 figures have been opened. Figures created through the pyplot interface (`matplotlib.pyplot.figure`) ar  
e retained until explicitly closed and may consume too much memory. (To control this warning, see the rcP  
aram `figure.max_open_warning`).  
max_open_warning, RuntimeWarning)
```



```
In [ ]:
```